

10.2 Operations with Matrices



$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -2 & 2 \\ 2 & 4 \end{bmatrix} \quad B + A =$$

$$A - B = \begin{bmatrix} 4 & -6 \\ -2 & 2 \end{bmatrix} \quad B - A$$

$$5A = \begin{bmatrix} 5 & -10 \\ 0 & 15 \end{bmatrix}$$

adding or subtracting matrices:

add or sub. each corresponding element
addition \rightarrow commutative

* Same order

scalar multiplication:

mult. each term by a constant

Properties of Matrix Addition and Scalar Multiplication

Let A , B , and C be $m \times n$ matrices and let c and d be scalars.

1. $A + B = B + A$ Commutative Property of Matrix Addition
2. $A + (B + C) = (A + B) + C$ Associative Property of Matrix Addition
3. $(cd)A = c(dA)$ Associative Property of Scalar Multiplication
4. $1A = A$ Scalar Identity Property
5. $c(A + B) = cA + cB$ Distributive Property
6. $(c + d)A = cA + dA$ Distributive Property

Solve for X:

$$3X + A = B$$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$3X = B - A$$

$$B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix}$$

$$X = \frac{1}{3} \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{4}{3} & 2 \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$$

$$3 \times 2 \quad 2 \times 2 = 3 \times 2$$

$$AB = \begin{bmatrix} 3 + (-12) & -2 + 3 \\ -12 + 8 & 8 + (-2) \\ -16 + 0 & 10 + 0 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -16 & 10 \end{bmatrix}$$

$$BA =$$

Matrix Multiplication

$$\begin{array}{c} \mathbf{A} \quad \times \quad \mathbf{B} \quad = \quad \mathbf{AB} \\ m \times n \quad \boxed{} \quad n \times p \quad \rightarrow \quad m \times p \end{array}$$

Is matrix multiplication commutative?

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

↑
identity

$$AI = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$IA =$$

Identity matrix:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Properties of Matrix Multiplication

Let A , B , and C be matrices and let c be a scalar.

1. $A(BC) = (AB)C$ *Associative Property of Multiplication*
2. $A(B + C) = AB + AC$ *Distributive Property*
3. $(A + B)C = AC + BC$ *Distributive Property*
4. $c(AB) = (cA)B = A(cB)$ *Associative Property of Scalar Multiplication*

$$\text{Solve: } x_1 - 2x_2 + x_3 = -4$$

$$x_2 + 2x_3 = 4$$

$$2x_1 + 3x_2 - 2x_3 = 2$$

$$AX=B$$

$$[A|B]$$

$$\rightarrow [I|X]$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \downarrow \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 2 \end{bmatrix} \quad X$$

$$\begin{bmatrix} 1 & -2 & 1 & \vdots & -4 \\ 0 & 1 & 2 & \vdots & 4 \\ 2 & 3 & -2 & \vdots & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & x_1 \\ 0 & 1 & 0 & \vdots & x_2 \\ 0 & 0 & 1 & \vdots & x_3 \end{bmatrix} \quad X$$

